Stochastic Modeling and Simulations of Thermostatically Controlled Loads

M. Goldshtein[a], V. Chernyak[b], M. Chertkov[c]

[a] Moscow Institute of Physics and Technology (State University), Dolgoprudny, Moscow Region, Russia
[b] Department of Chemistry, Wayne State University, Detroit, MI, USA
[c] Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM, USA

Abstract

We study a stochastic model of an ensemble of Thermostatically Controlled Loads. The study includes direct Markov Chain simulations of the ensemble, validation of the results against the Fokker-Planck type of theory in the regimes amenable for analytics, and then numerical exploration of various non-equilibrium properties of the system in more challenging regimes, e.g. analyzing response to demand response perturbations of practical interest. This is an early report on the study aimed at designing new controls of the TCL ensembles.

The simplest model without control

The simplest model describes a behavior of a large number N of identical customers under the effect of the ambient. The state of a customer is characterized by the temperature, denoted by T. Stochastic dynamics for the temperature relaxation process is given by the Langevin equation:

$\dot{T} = -\frac{1}{\tau}(T - T_{out}) + \xi(T,t)$, where the Gaussian noise $\xi(T,t)$ is assumed be $\begin{cases} <\xi(t)> = 0 \\ <\xi(t_1)\xi(t_2)> = D\cdot\delta(t_1-t_2) \end{cases}$

The Fokker-Plank equation, established for such a system:

$$\begin{cases} \dot{\rho} = \frac{1}{\tau}\partial_T[(T - T_{out})\rho] + \frac{D}{2}\partial^2_{TT}\rho, \\ \rho(T,0) = \delta(T - T_0) \end{cases}$$

The solution:

$$\rho(T,t) = \frac{1}{\sqrt{\pi D\tau}}\cdot\frac{1}{\sqrt{1-e^{-2t/\tau}}}\cdot\exp\left(-\frac{\left((T-T_{out})+(T_{out}-T_0)e^{-t/\tau}\right)^2}{D\tau\left(1-e^{-2t/\tau}\right)}\right).$$

The simple iteration method in application to the Langevin equation: $T_{i+1} = T_i - \frac{dt}{\tau}(T_i - T_{out}) + \xi\sqrt{dt}.$

Two states control modelling

In the two states control modeling the state of a customer is described by the inside temperature, denoted by T and a discrete variable j that attains values j = 2 and j = 1 for the device (air conditioner) being off and on, respectively. Each regime is given by the Langevin equation aforementioned:

$$\dot{T} = -\frac{1}{\tau}(T - T_j) + \xi(T,t),$$

where T_2 and T_1 are the outside temperature and the inside temperature for permanently working air conditioner. We set dead-bands by initiating the bounds of the comfort zone T_{down} and T_{up}. Achieving one of them we change over the conditioner program.

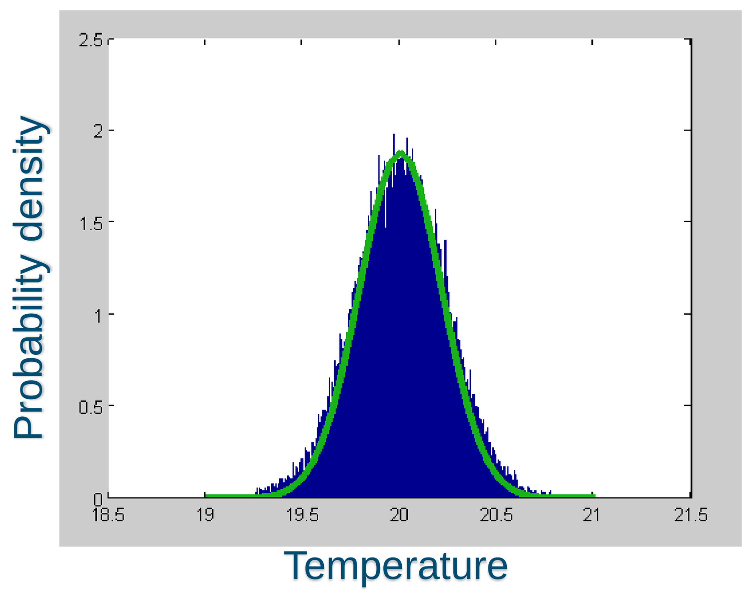
The system of Fokker-Plank equations, established for such a model:

$$\begin{cases} \frac{\partial\rho_1}{\partial t} = \frac{D}{2}\frac{\partial^2\rho_1}{\partial T^2} - \frac{\partial}{\partial T}(f_1(T)\rho_1) - r_{21}(T)\rho_2 + r_{12}(T)\rho_1 \\ \frac{\partial\rho_2}{\partial t} = \frac{D}{2}\frac{\partial^2\rho_2}{\partial T^2} - \frac{\partial}{\partial T}(f_2(T)\rho_2) - r_{12}(T)\rho_1 + r_{21}(T)\rho_2 \end{cases},$$

where $\begin{cases} f_j = -\frac{1}{\tau}(T - T_j) \\ r_{12}(T) = r\cdot\theta(T - T_{up}) \\ r_{21}(T) = r\cdot\theta(T - T_{down}) \end{cases}$

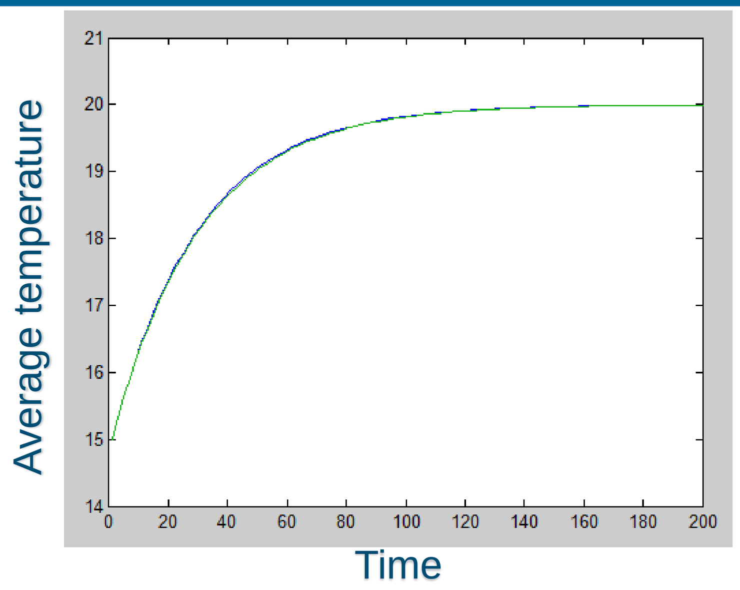
We use $r \to \infty$ limit, because it is a correct way to describe the situation of instantaneous switching, once out of range.

Results of the simplest model simulations



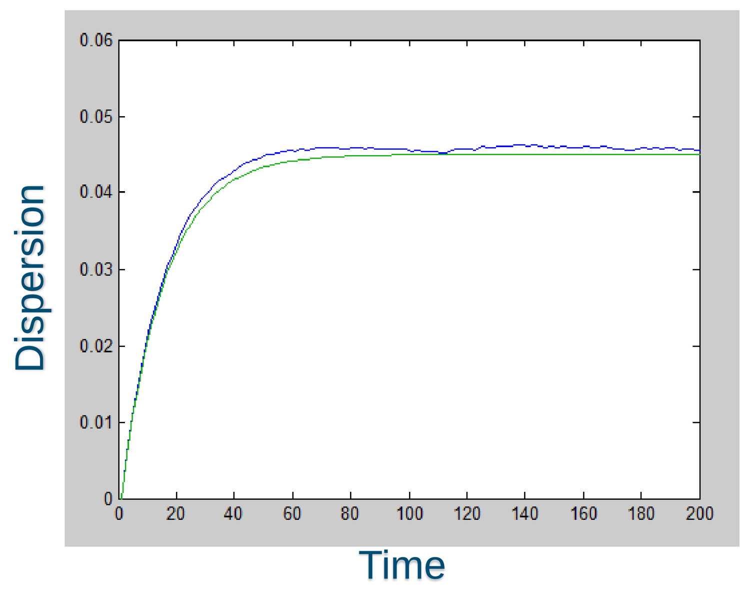
The stationary temperature distribution

Green line – solution to the Fokker-Plank equation



The average temperature dependence on time.

Blue line – simulation result
Green line – theoretically established function
$T_{average}(t) = T_{out} - (T_{out} - T_0)e^{-t/\tau}$



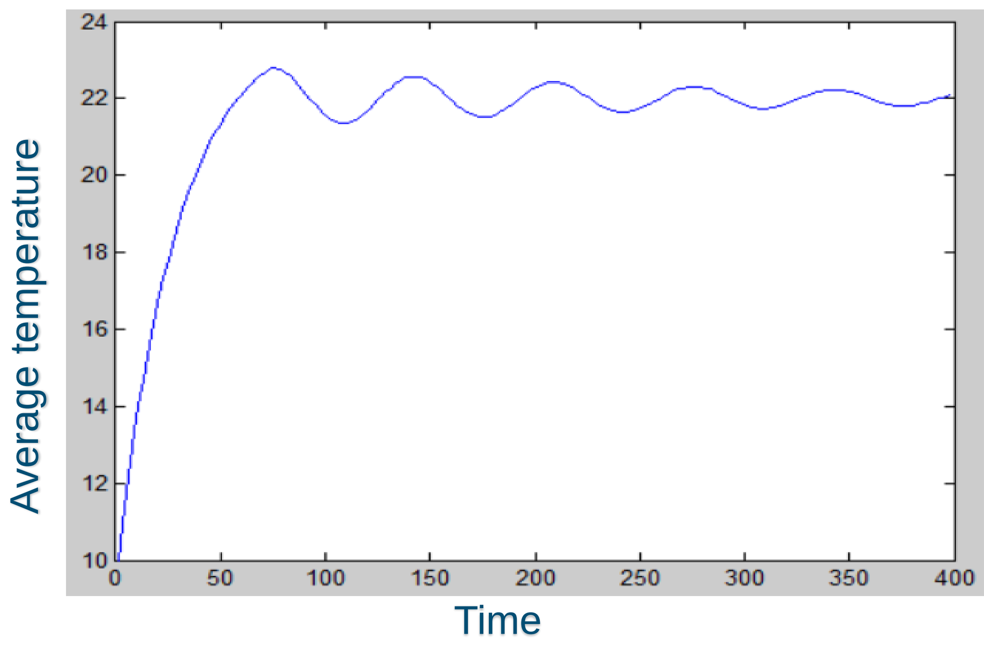
The dispersion dependence on time.

Blue line – simulation result
Green line – theoretically established function
$\sigma(t) = \frac{D\tau}{2}(1 - e^{-2t/\tau})$

Simulation parameters: N = 50 000; Mtime = 1000 (dt = 0.01); D = 0.3; $\tau = 0.3$; $T_{out} = 20^0C$; $T_0 = 15^0C$

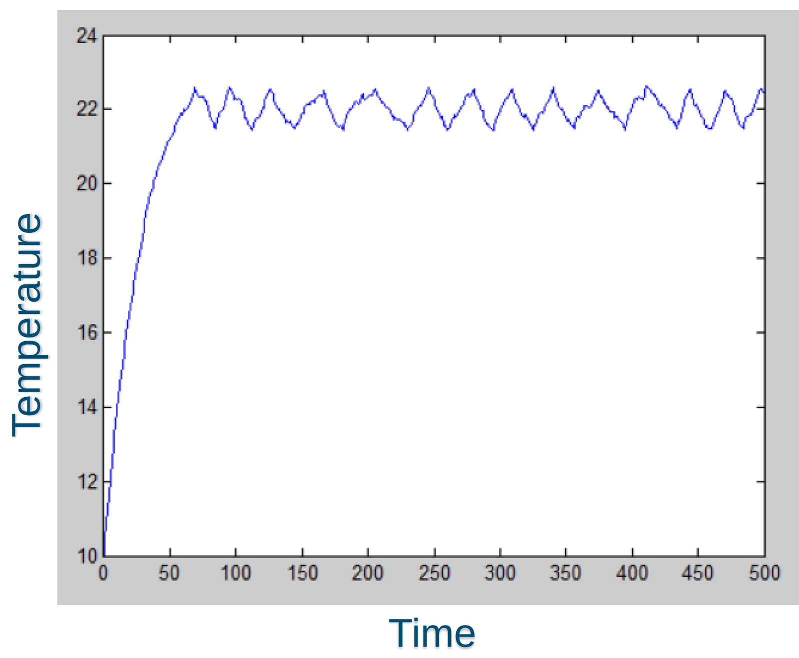
A range of parameters, such as D, τ, T_{out}, N and Mtime, effect the temperature relaxation process. The more D or τ are, the wider the temperature distribution is and the more the stationary dispersion value is. The less dt is, the closer to reality we are, but the more time steps we have to do to achieve the stationarity so the more the time of calculations is. The diminution between T_{out} and T_0 influences the relaxation time.

Results of the two states control model simulations



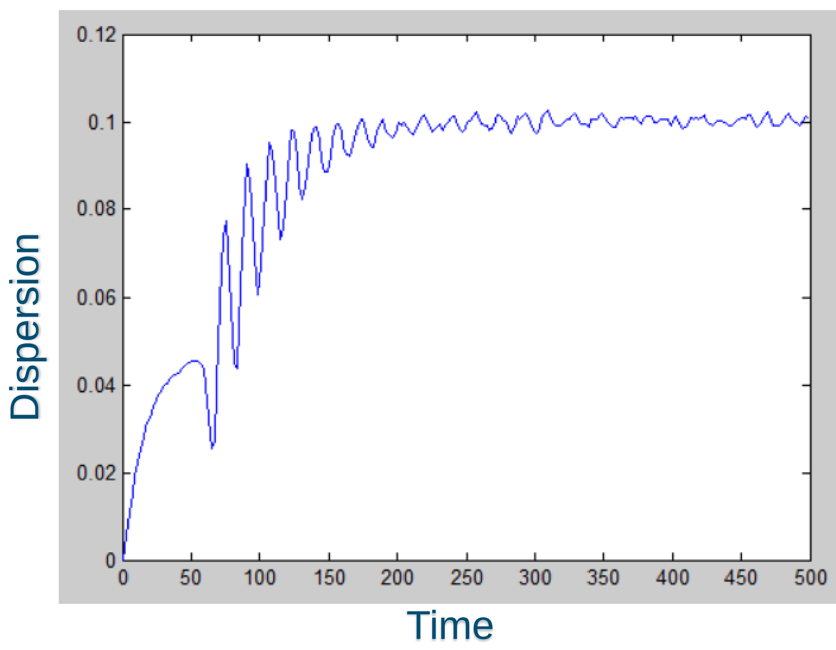
The average temperature dependence on time.

Perturbations are quite high at the beginning, but they are decreasing in time.



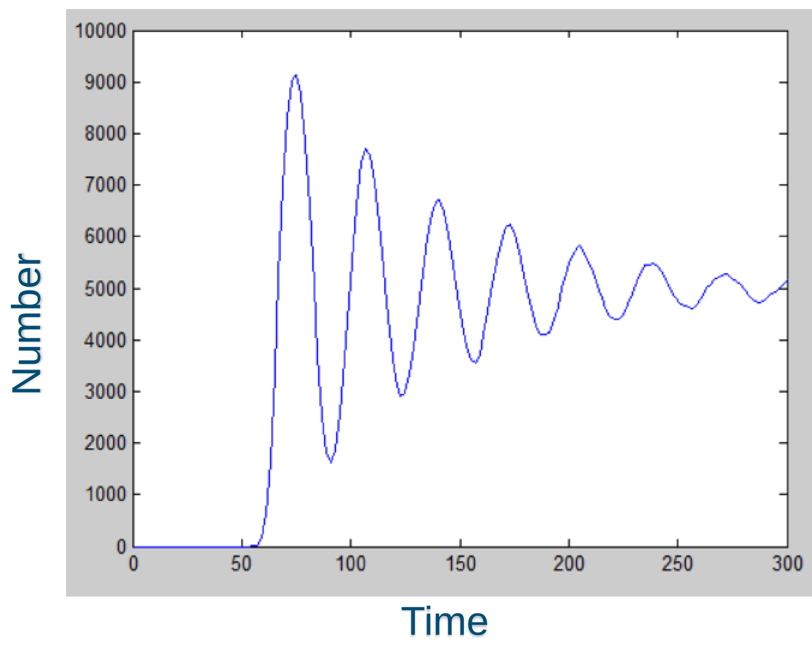
The temperature of a random customer on time.

Because of control the customers go back and forth between dead-bands passing round.



The dispersion dependence on time.

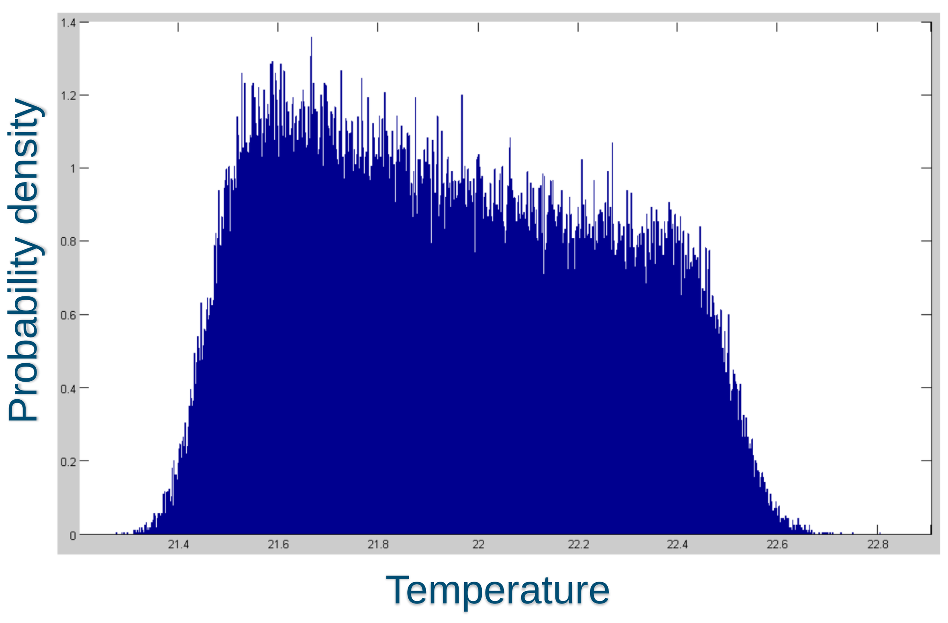
While all customers are switched off/on, the dispersion is as the same as in the simplest case without control. Perturbations of the dispersion are decreasing in time quite slowly.



The number of ON-state customers on time.

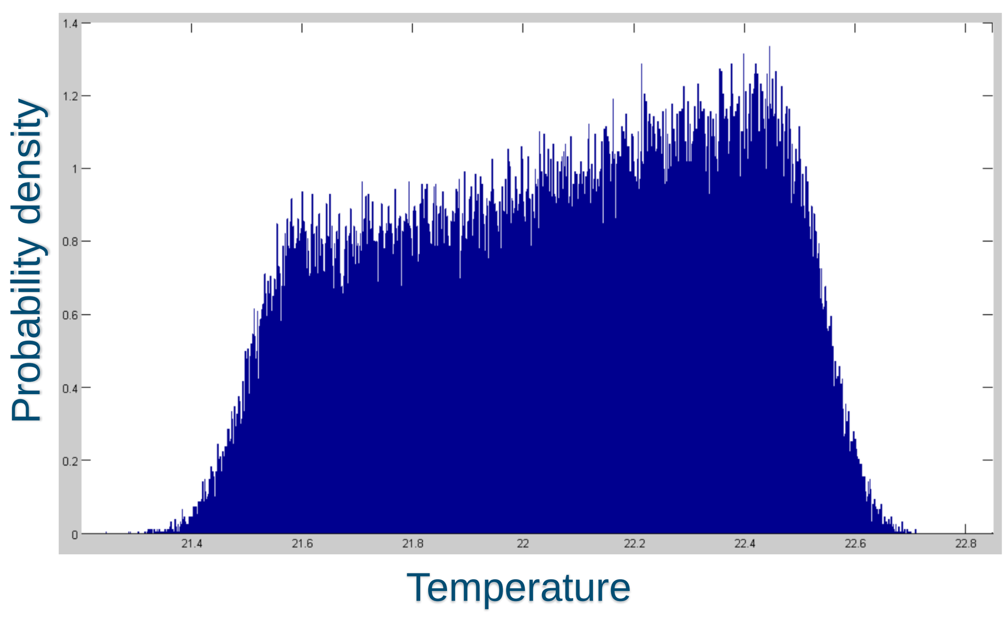
Because of noise perturbations are decreasing in time and approaching to N/2. If we set, for example, heating force bigger than the freezing one, the stationary number will be bigger.

Simulation parameters: N = 100 000; Mtime = 1000; dt = 0.01; D = 0.3; $\tau = 0.3$; $T_0 = 10^0C$; $T_1 = 20^0C$; $T_2 = 24^0C$; $T_{down} = 21^0C$; $T_{up} = 23^0C$



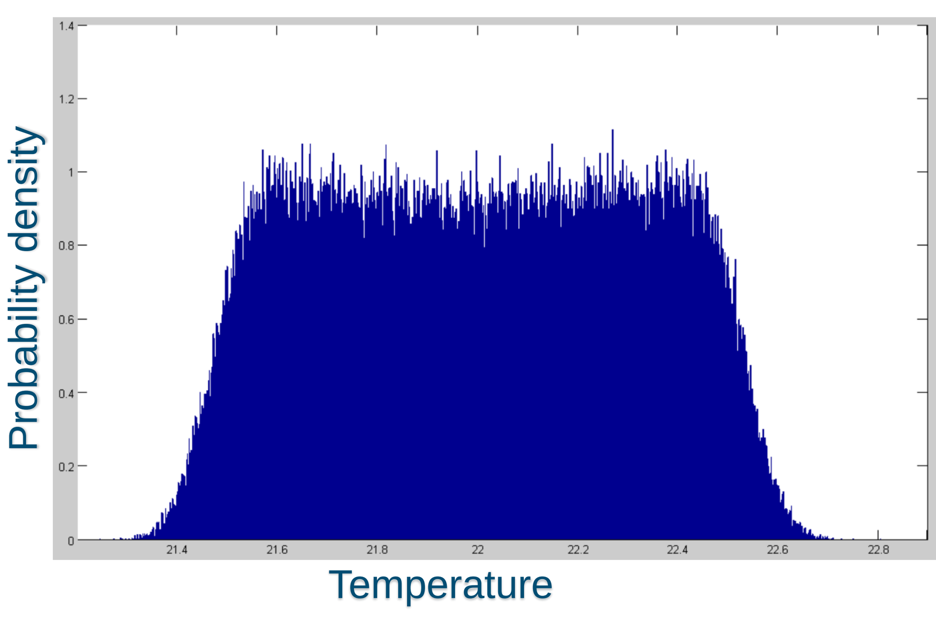
The stationary temperature distribution for ON-state customers

The peak is shifted to the left because the freezing force is getting less effective with the temperature decreasing.



The stationary temperature distribution for OFF-state customers

The peak is shifted to the right because the heating force is getting less effective with the temperature increasing.



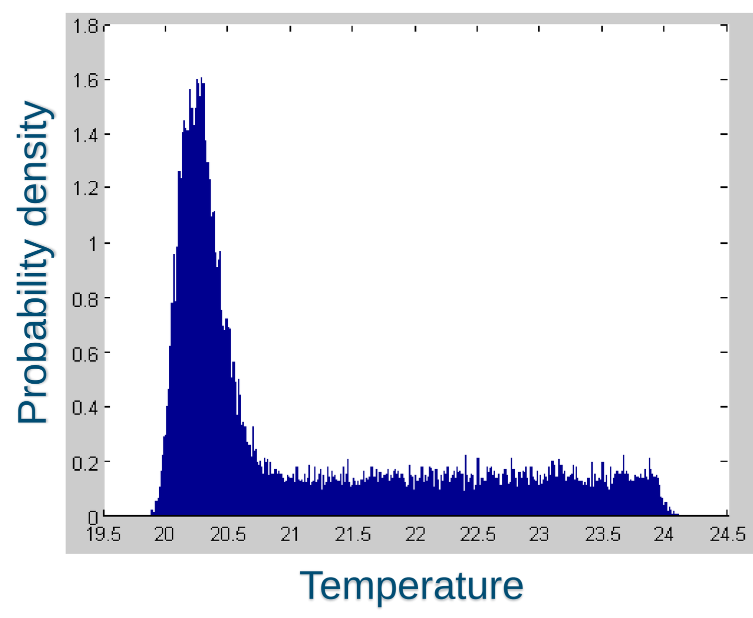
The stationary temperature distribution for all customers

Simulation parameters

N = 200 000; Mtime = 500 (dt = 0.01)
D = 0.3; $\tau = 0.3$

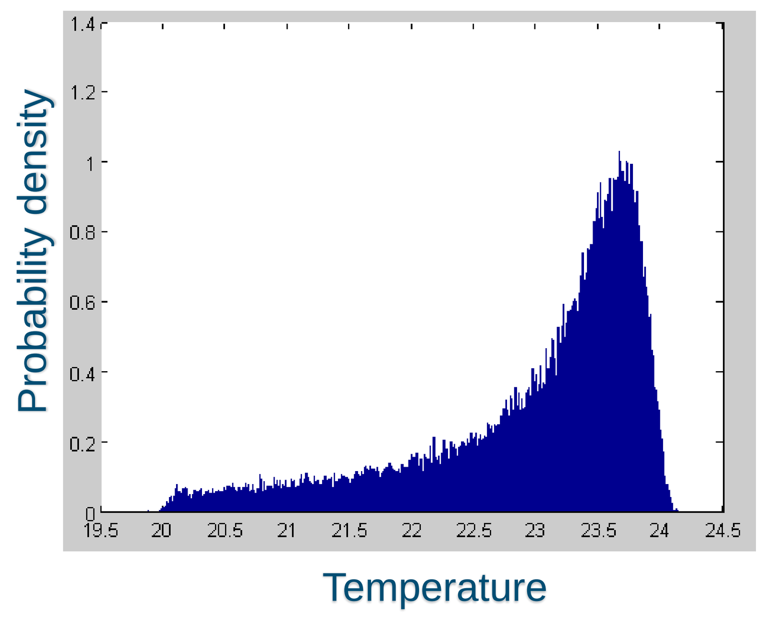
$T_0 = 10^0C$
$T_1 = 20^0C$; $T_2 = 24^0C$
$T_{down} = 21^0C$; $T_{up} = 23^0C$

The probabilities of having any temperature from 21.6℃ to 22.4℃ are the same. The character of distribution tails is effected by noise.



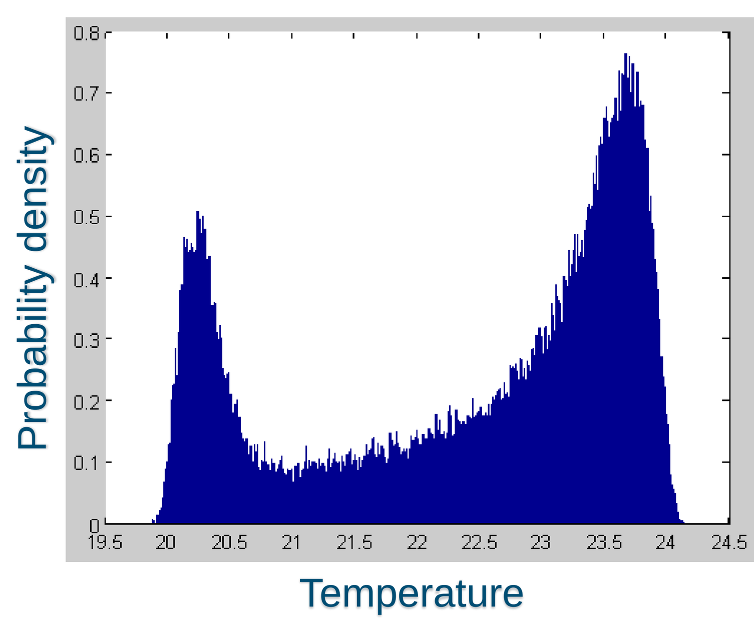
The stationary temperature distribution for ON-state customers

The freezing force is negligible close to T_1.



The stationary temperature distribution for OFF-state customers

The heating force is negligible close to T_2.



The stationary temperature distribution for all customers

Simulation parameters

N = 200 000; Mtime = 500 (dt = 0.01)
D = 0.3; $\tau = 0.3$

$T_0 = 10^0C$
$T_1 = 20^0C$; $T_2 = 24^0C$
$T_{down} = 20^0C$; $T_{up} = 24^0C$

The closer to dead-band the temperature is, the more important the role of noise becomes.

Conclusion and plans for future

We have researched some system properties concerning to stationary temperature distributions and some time-variable characteristics in different regimes. These results are preliminary. In the nearest future we are planning to validate our results theoretically. The system behavior researching brings us to creation of an algorithm of demand-response control.